DEVELOPMENT OF UNIPASS – A UNIFIED PROBABILISTIC ASSESSMENT SOFTWARE SYSTEM

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Abstract

A dramatic advancement in probabilistic assessment tools, ***UNIPASS™*** offers users a comprehensive software system for generating probabilistic analysis. Combining a familiar Windows-based interface with a simple menu-driven system, ***UNIPASS*** integrates easily into most existing environments without a significant learning curve or additional hardware investment. It also takes advantage of proven methodologies to yield consistent and reliable results, and generates several types of analyses according to user-customizable parameters. This paper brief s its many features, options and capabilities of UNIPASS software.

2. INTRODUCTION

Today's technology companies face some daunting challenges. Whether their need is to improve designs, optimize maintenance, improve reliability or compete more effectively, all of these translate to save time, materials and money and, therefore, directly to that company's bottom line. Solving problems quickly, efficiently, and completely can mean a program's success or failure – but how can a company ensure that they have everything they need to make informed decisions and get the best results?

UNIPASS™, a **UNI**fied **P**robabilistic **A**ssessment **S**oftware **S**ystem is a general-purpose probabilistic program to help organizations meet and conquer the challenges. It contains a user-friendly Graphical User Interface (GUI), state-of-the-art probabilistic analysis techniques (including several unpublished proprietary methods), a large library of statistical distributions, and a script type function module with a large library of support functions that can easily define any limit-state function in a "scripting FORTRAN-like syntax" format [1]. The scripting type of limit-state function allows users to execute ***UNIPASS*** without recompiling the program for different problems.

UNIPASS can operate either as a stand-alone program or as the core of an integrated software environment that incorporates an unlimited amount of third-party software. Because ***UNIPASS*** works with the native input/output file format of external codes, in terfacing ***UNIPASS*** w ith **MSC/NASTRAN™** or any other software requires minimal effort. Furthermore, ***UNIPASS*** can detect the effect of a particular random variable to minimize the number of external software calls. This versatility allows virtually unrestricted freedom in introducing random variables into many practical analyses that, until now, have been strictly deterministic.

But ***UNIPASS*** is more than just an analysis tool. Its inverse probability analysis and sensitivities analysis capabilities make it a powerful design aid in any product cycle. ***UNIPASS*** is equipped with advanced artificial intelligence that is designed to handle systems with an essentially unlimited number of random variables and distribution types with ease and efficiency. Its modular arrangement allows you to tailor an analysis to the desired level of accuracy and efficiency.

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In most of engineering applications, each limit-state function calculation requires ***UNIPASS*** to interface with one or more time-consuming external software packages (e.g., finite element software). Consequently, the calculation of limit-state functions expends the majority of CPU resources. Therefore, the efficiency of an analysis is measured by the number of limit-state function calculations.

This paper briefly describes its many features, options and capabilities. Users will benefit from its outstanding functionality, and from the support and expertise of its manufacturer, UNIPASS Technologies, Inc. [2, 3].

2. FOUR PROBLEM TYPES

In probabilistic analysis, the failure probability of a system may be expressed in the form

$$Pf = \text{Prob}\left[\bigcup_{k}\bigcap_{j\in C_k}\left\{g_j(\underline{x})\leq 0\right\}\right] \qquad (1)$$

where $g_j(\underline{x})$ is the j-th limit-state function expressed in terms of the vector of random variables $\underline{x}$; C_k denotes the k-th cut set representing a set of limit-states, the joint symbol indicates a joint exceeding of the constraints in each cut set constitutes the failure of the system, and the union symbol shows the system failure probability is the union of failure probability over all of the cut sets.

Problem types are categorized according to the number of cut sets and the number of limit-state functions in each cut set. ***UNIPASS*** can perform probabilistic analysis for four different problem types, as described below.

Component Problems: Model that contains a single cut set, which contains only a single limit-state function.

Serial System Problems: Models that are defined by two or more cut sets, with each cut set containing a single limit-state function.

Parallel System Problems: Models that contain a single cut set, which comprises more than one limit-state function.

General System Problems: Models that are a combination of serial and parallel systems.

3. THREE ANALYSIS TYPES

UNIPASS can perform three analysis types, as listed below.

Probability Analysis: This is the conventional reliability analysis as expressed in Eq. (1) that calculates the probability, the associated sensitivities and the most likely conditions (MPP) for a specified level in each limit-state function.

Inverse Probability Analysis: This analysis identifies the limit-state function level and the most likely conditions that will produce a predefined probability. The associated sensitivities of probability with respect to random variables and distribution parameters are also calculated in this analysis. The inverse probability analysis can only be performed on component probability problems.

PDF/CDF Analysis: This analysis calculates the predefined range of probability density function (PDF) and cumulative distribution function (CDF) of the limit-state function with the minimum or fixed number of points in the CDF/PDF curve. This analysis can be only applied to component probability problems.

4. SIX PROBABILISTIC METHODS

UNIPASS provides six major categories of probabilistic methods:

- **First Order Reliability Methods (FORM)**
- **Second-Order Reliability Methods (SORM)**
- **Simulation Methods (SM)**
- **Importance Sampling Methods (ISM)**
- **Response Surface Methods (RSM)**
- **Mean Value-Based Methods (MVBM)**

These six probabilistic methods can be applied in Probability, Inverse Probability, CDF/PDF Analyses for the component/system problems. A summary of the available analyses is listed in Table 1.

4.1 First Order Reliability Methods (FORM)

In FORM, probability approximations are made by replacing each limit-state surface with a first-order polynomial approximation of the limit-state function at a point on the failure boundary. This point is usually the point nearest to the origin in a transformed standard normal space, and is generally referred to as the design point, maximum likelihood point or most probable point (MPP).

4.2 Second Order Reliability Methods (SORM)

In SORM, probability approximations are made by replacing each limit-state surface with a second-order polynomial approximation of the limit-state function at the corresponding MPP. ***UNIPASS*** offers two approaches for SORM: Curvature-Fitting and Point-Fitting methods. It is noted that Point-Fitting method (see Figure 1) is implemented by fitting a piecewise paraboloid surface to discrete points on the limit-state surface around the MPP in the standard normal space. The analytical formulation of second-order failure probability is available for component reliability problem while for system reliability problems, the simulation methods can be used in conjunction with second-order approximation limit-state functions.

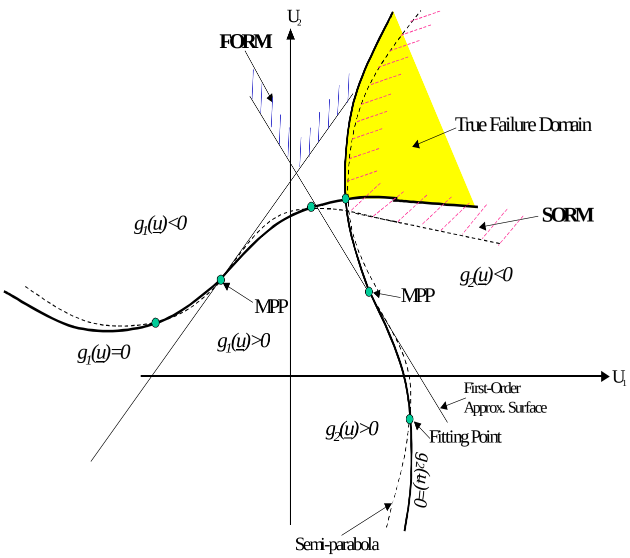

Figure 1 Example of the SORM Point-Fitting Method

4.3 Simulation Methods (SM)

UNIPASS offers three simulation methods: Monte Carlo, Latin Hypercube and Directional Simulation. These methods give the exact result if the true limit-state function is used and the sampling points are sufficiently large. Simulation methods can also be called for probability calculations when the limit-state functions are approximated by FORM, SORM, RSM or MVBM.

Monte Carlo Simulation: In this method, deterministic analysis is carried out for a series of sample points (a set of random variables) generated in accordance with their probability distributions. The desired statistics of the response quantities, such as the mean, variance and probabilities, are then evaluated based on the generated samples. The sensitivity of failure probability, with respect to the distribution parameters and the PDF/CDF of the limit-state function, is also available with no additional limit-state function calculation. Both point and interval estimators of failure probability may be used in Monte Carlo simulation.

Latin Hypercube Simulation: In this method, the sampling space is divided into subsets of equal probability. However, the number of samples is reduced to represent each subset of each independent random variable only once. Only the point estimator of failure probability is given in this method.

Directional Simulation: In this method, uniform sample directions are generated. The intersections of these directions with the limit-state functions are then identified and used to calculate probabilities. Both the point and interval estimators of failure probability are given in this method.

4.4 Importance Sampling Methods (ISM)

UNIPASS offers two different Importance Sampling Methods: Sphere-based and Directional Simulation. In these methods, sampling density is skewed toward the MPP.

Sphere-based Importance Sampling Method: The sampling points located inside the sphere with the predefined radius R (in standard normal space) are assumed to be in the safe domain. Therefore, no limit-state function calculation is needed for these sampling points. Only the sampling points that are outside the preceding sphere are used to calculate the limit-state function values. The most appropriate radius of the sphere should be less than the generalized reliability index β, which is output of FORM and SORM. The efficient of this method reduces as the number of random variables increases. Both the point and interval estimators of failure probability are given in this method.

Directional Importance Sampling Method: The unit direction sampling is concentrated around the MPPs as an example shown in Figure 2. Both the point and interval estimators of failure probability are given in this method.

4.5 Response Surface Methods (RSM)

In RSM, a series of experiments is performed to calculate the true limit-state function at several points in the design space. Utilizing techniques such as the least-squares method, the results of these experiments will be used to approximate the true limit-state surface with a second-order polynomial. ***UNIPASS*** produces these experiments using the Box Behnken Method. Expansion points for Box Behnken matrices may use mean point or any user-defined point.

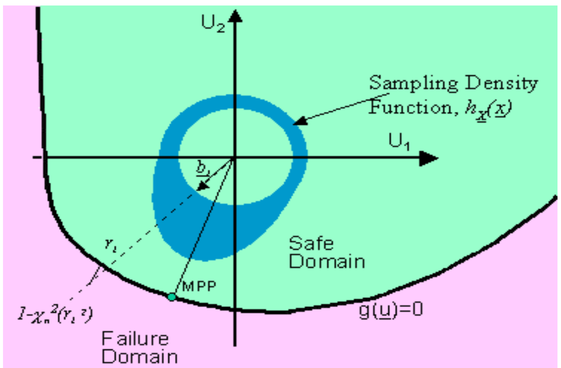

Figure 2 Example of Directional Importance Sampling Method

4.6 Mean Value-Based Methods (MVBM)

MVBM involves first-order Taylor series expansion of limit-state functions around the mean values of the random variables. ***UNIPASS*** offers three different methods in this category:

- Mean-Value First-Order Second-Moment Method (MVFOSM)
- Mean Value Method (MV)
- Advanced Mean Value Method (AMV)

Either the mean value method or the advanced mean value method can be applied in the original and standard normal space. The primary purpose of the mean-based methods is to compute the first and second moments (i.e., mean and standard deviation) of the limit-state function. For failure probability, satisfactory results can only be obtained with these methods when the random variables do not deviate significantly from normal and most importantly, when the limit-state function is not highly non-linear in terms of random variables.

5. ELEVEN MPP IDENTIFICATION METHODS

The key to efficient and stable first-/second-order reliability analysis is in the speed and accuracy of identifying one or more MPPs. The MPP is defined as the highest density point in the failure domain. The U-based MPP is defined as the minimum distance point of the failure boundary to the origin in the standard normal space. Therefore, the MPP is also known as the most likely failure point and the design point. This point is important because the majority of failure probability is contributed from the probability density of the failure domain around the MPP.

The MPP identification problem is formulated as a constrained optimization. Algorithms for this class of problem have been extensively investigated. However, it has been recognized that different problems require different optimization procedures in order to avoid erroneous solutions and lack of convergence.

Most optimization techniques for MPP identification require the use of derivatives of the limit-state function with respect to the random variables. This, in turn, requires the limit-state function to be continuous and differentiable. However, sometimes these requirements are not satisfied, or the first-order derivatives of the limit-state function are very difficult to calculate accurately. The inherent difficulties resulting from numerical round-off errors are more pronounced in probabilistic problems commonly encountered in the industry. ***UNIPASS*** has implemented a non-gradient-based MPP search algorithm that eliminates the dependency on the aforementioned requirements. Altogether, ***UNIPASS*** offers the following 11 different iteration procedures for MPP identification.

- U-based/U-linearized MPPL method
- Modified U-based/U-linearized MPPL method
- U-based/X-linearized MPPL method
- Modified U-based/X-linearized MPPL method
- HL-RF method
- Modified HL-RF method
- Improved HL-RF method
- Sequential Quadratic method
- Gradient Projection method
- Modified Gradient Projection method
- Simulation Search method (Non-Gradient-based method)

6. THIRTY SEVEN PROBABILITY DISTRIBUTIONS

UNIPASS has a library of 37 different probability distributions that may be used to define 4 different classes of random variables. The 37 distributions are listed below (plus truncated distributions) [4].

Deterministic	Beta
Chi-square	Double Exponential
Exponential	F distribution
Gamma	Gumbel

Logistic	Lognormal
Maxwell	Normal
Pareto	Rayleigh
Student t	Triangular
Truncated Normal	Type I smallest
Type II largest	Uniform
Weibull	User Data

For each distribution type, the precision of PDF, CDF and inverse CDF calculations have been tested to be accurate up to 14 digits with a wide range of CDF values from 2.8×10^{-55} to 0.9999999999999 (i.e., $\Phi[-15.615]$ to $\Phi[7.385]$ where Φ is the CDF of the standard normal distribution.

6.1 User's Defined Distribution

User defined distribution is called 'User Data' distribution. This distribution is used when the user wants to input a random variable with a distribution other than the "known" types of statistical distributions used in ***UNIPASS***. The random variables with user defined distribution can also be co-dependent with other random variables. Four options are available for creating a new user defined distribution. PDF and CDF data points can be typed in manually or imported from a text file. The data points are sorted as x and y pairs with x being the abscissa and y being the PDF or CDF value at x. Any number of numeric values can be placed in a row. However, when a distribution is save to the buffer, the x, y pairs are sorted in ascending order and stored in its own row.

An example of user defined distribution is given below. Assume the random variable has its PDF defined by the following 9 points:(0.0, 0.0), (0.25, 0.5), (0.5, 0.99), (0.75, 0.5), (1.0, 0.0), (1.25, 0.5), (1.5, 0.99), (1.75, 0.5), (2.0, 0.0). The input would be shown as in Figure 3 and Figure 4.

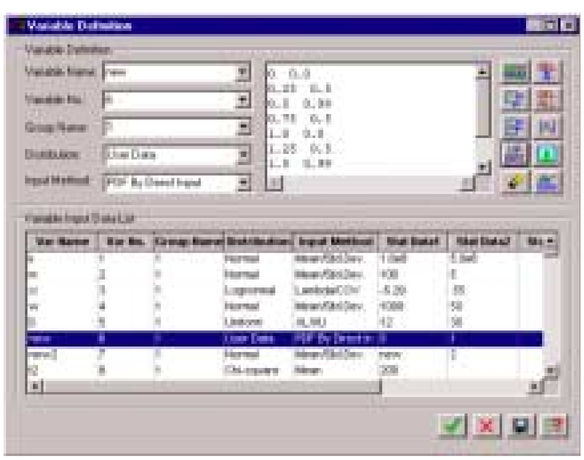

Figure 3 Example of User Defined Distribution

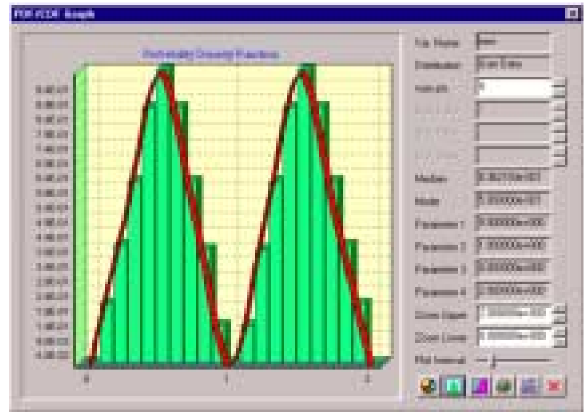

Figure 4 PDF plot for the User Defined distribution defined in Figure 3

7. FOUR CLASSES OF RANDOM VARIABLES

UNIPASS provides four classes of random variables. These are listed below [2, 3].

Class 1, Independent Random Variables

The continuous random variables X and Y are said to be statistically independent random variables, if a change in X does not produce any change in Y.

Class 2, Dependent Random Variables with Correlation Matrix

The random variables X and Y are said to be dependent with correlation coefficient (or dependent with incomplete information) if the correlation coefficient is available and the complete probability law is not. In this case, the correlation matrix is entered in the Correlation Matrix window. ***UNIPASS*** uses the Nataf Probabilistic Transformation method to calculate the corresponding correlation matrix in the standard normal space.

The Correlation coefficient Matrix is a symmetrical, positive definitive matrix with 1 in diagonal entries and values between –1 and 1 in the off diagonal entries. Therefore, only the lower or upper triangle of the matrix requires input values. After finishing the entry, click the OK button to leave this window. Use the Plot Joint PDF button to plot the joint PDF (see Figure 5). Use the scroll bars in the upper right-hand corner to rotate the view angle.

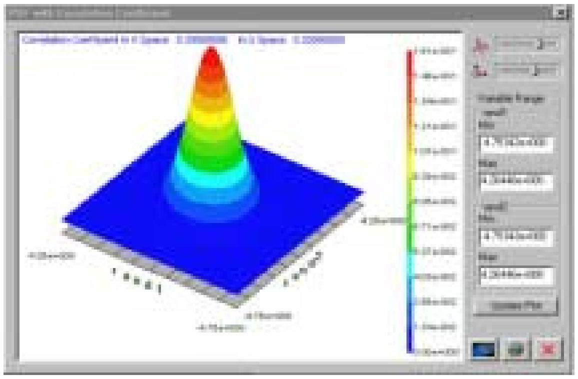

Figure 5 Joint PDF Plot for variables for Rand1 and Rand2

Note that the minimum and maximum values of Rand1 and Rand2 are provided in the edit boxes on the right-hand side. By changing these values in the edit boxes, the plotting range can be modified. A zoomed view is provided in Figure 6.

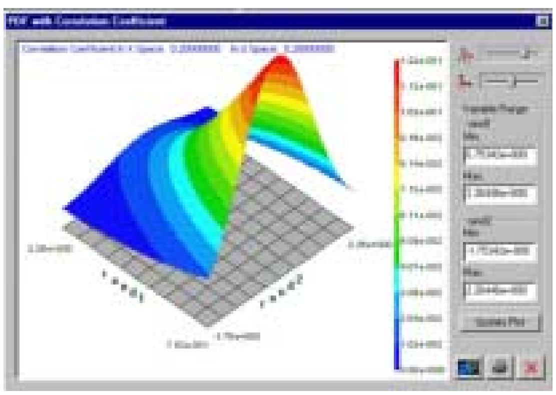

Figure 6 A Zoomed View of the Same Joint PDF Shown in Figure 5

Class 3 and 4, Random Variables with Hyper-Parameters (Conditional Distribution)
If there are two or more random variables, the mean and variance (or parameters) of one variable may depend on the value(s) of the other random variable(s). In such cases we have conditional means, conditional variances or conditional parameters.

- **Class 3:** mean, variance or parameters belong to Class 1 type random variables.
- **Class 4:** mean, variance or parameters belong to Class 2 type random variables.

The PDF plot of the conditional variable is presented in the joint PDF of the conditional variable and one random variable used in the hyper-parameters. The marginal PDF of the conditional random variable with a specified value of random variable for the hyper parameter can also be shown in the same figure of the preceding joint PDF. The marginal PDF of random variable Rand4 by a fixed value of Rand1 is given by clicking the desired value of Rand1 along the axis of Rand1. One example is given Figure 7.

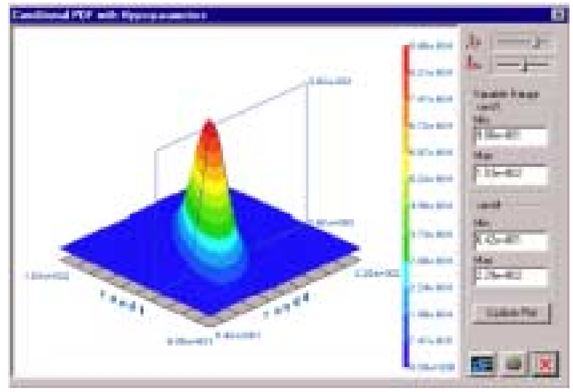

Figure 7 The Joint PDF of Rand4 and Rand1 and marginal distribution of Rand4

8. SCRIPTING TYPE LIMIT-STATE FUNCTION INPUT

UNIPASS allows the users to define the limit-state function by using FORTRAN-like syntax. This capability allows the user to run ***UNIPASS*** without recompiling the program for different problems. The limit-state function may be defined as a function of random variables or previously defined functions. These functions may also be a function of random variables or other previously defined functions.

UNIPASS provides the following three interfaces with other codes:

- NASTRAN Interface
- Customized interface
- Generic interface

UNIPASS also has a tracking capability for determining the effect of random variables on the results of the external software. This tracking capability enables ***UNIPASS*** to minimize the number of external software calls.

9. INTERFACES WITH MSC/NASTRAN

UNIPASS is distributed with an intuitive Graphical User Interface (GUI) with the **MSC/NASTRAN™** finite element code. The integrated package can incorporate probabilistic mechanics into every structural analysis offered by **MSC/NASTRAN**[5]. Typical response values are:

response type:	nodal displacement element stress element strain element force
response location:	specific node or element number maximum response value minimum response value maximum absolute response value

10. CUSTOMIZED INTERFACE

This interface allows simultaneous integration with one or more commercial and/or in-house software tools. In an integrated software environment, ***UNIPASS*** provides the core engine for modeling uncertainties, computing probabilities and performing sensitivity analysis, while other deterministic software tools provide the computational framework for evaluating the limit-state functions. The ***UNIPASS*** Customized Interface handles the majority of work required to establish communication between various software packages [6].

The only task handled by the user is the development of an interface program to extract the desired data from the output of deterministic software tools. Furthermore, the user can use random variable names and random function names as design parameters in the input file of deterministic software tools. Users then compile this interface as a dynamic link library file (DLL) and replace the existing corresponding file.

11. GENERIC INTERFACE

Without requiring users to do any programming work such as the DLL in the Customized Interface, ***UNIPASS*** Generic Interface may be simultaneously integrated with an unlimited number of deterministic in-house and/or commercial software tools. In the integrated software environment, ***UNIPASS*** provides the core engine for modeling uncertainties, computing probabilities, and performing sensitivity analysis; while the deterministic software tools provide the computational framework for calculating limit-state function [2].

UNIPASS Generic Interface is designed to connect with any program with text input/output. A response value is extracted from a text output file through a sequence of search command. A search command can be based on a keyword/key phrase, or relative location with respect to the row marked by the last search command. Each search sequence can have any number of keyword or location commands in any order. However, the last search command of any sequence must be a location command.

The EXTERNAL.IN file is required when the limit-state function calls the interface with command line software and text input/output files. The external software is invoked by the ***UNIPASS*** support function EXTERNAL in the Function Definition window. Upon clicking EXTERNAL.IN in the Create drop-down menu, the window in Figure 8 appears:

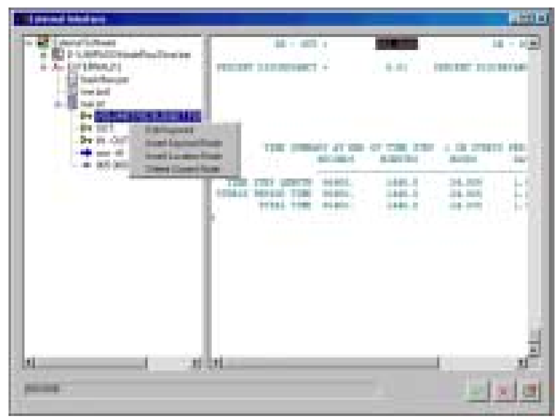

Figure 8 Generic interface definition window

12. ANIMATION SOLVER WINDOW

UNIPASS Solver is driven by a batch file, named UNIPASS.IN. This file can be prepared by UNIPASS Pre-processor (GUI) or by any test editor [7]. Users can view the progress of analysis after launching ***UNIPASS*** solver as an example shown in Figure 9. In this example, a Monte Carlo simulation was carried out and the PDF of limit-state function was animated in the window as the solver runs. User time elapsed is also updated automatically during the analysis. On NT 4.0 or later, you can also monitor the actual CPU time allocated to the solver computation thread.

13. SENSITIVITY MEASUREMENTS

The analysis of uncertainty involves measuring the degree to which each random variable and its distribution parameters contribute to the uncertainty in the output quantities (limit-state function, failure probability and reliability index). Generally, sensitivity analysis is a byproduct of probability calculation such that no extra limit function calculation is required.

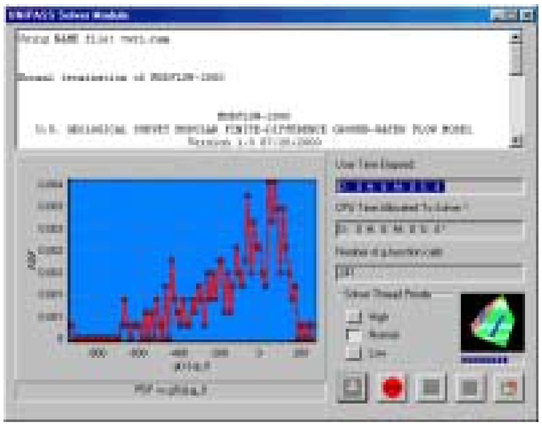

Figure 9 Window Showing UNIPASS Solver Started

Because the limit-state function is expressed either explicitly or implicitly in terms of random variables, the resulting failure probability is also a function of random variables. Furthermore, because the probability transformation from the original space to the standard normal space employs distributions of random variables, the standard normal variates are functions of distribution parameters, and the limit-state function in the standard normal space is also a function (implicitly) of distribution parameters. Therefore, the sensitivities of the reliability index and failure probability with respect to random variables and their distribution parameters are available in the analysis.

In addition, the sensitivity of output quantities with respect to other constant parameters may be of interest. These sensitivities can be obtained using ***UNIPASS*** by specifying these constants as *deterministic random variables*. In ***UNIPASS***, a failure function (or limit-state) can be formulated with any number of these deterministic random variables alongside other random variables.

The following are the typical sensitivity measurements provided by ***UNIPASS***:

- Sensitivity of limit-state function with respect to the random variables at the MPP in both the standard normal space and the original space.
- Sensitivity of failure probability and reliability index with respect to random variables.
- Sensitivity of failure probability and reliability index with respect to means and standard deviations.
- Sensitivity of failure probability and reliability index with respect to distribution parameters.
- Dimensionless sensitivity of failure probability and reliability index with respect to random variables, means and standard deviations.

14. <u>OUTPUT FILE HTML VIEW</u>

This menu item parses the output file into a hyper linked HTML file and loads the file into the main viewer. Users can click any entry in the Table of Content of HTML file to scroll the main viewer to a specific section. Conventional Web Browser Navigational Controls such as backward, forward, stop, refresh are built to help users.

An example of UNIPASS HTML output file is given in Figure 10 which has the Table of Content as shown below.

UNIPASS SOLVER OUTPUT CONTENTS

- Problem Definition
- MPP Identification
- Summary of MPP Identification for CDF/PDF
- Probability Calculation
- Summary of CDF Analysis
- Summary of PDF Analysis
- Sensitivity Calculation

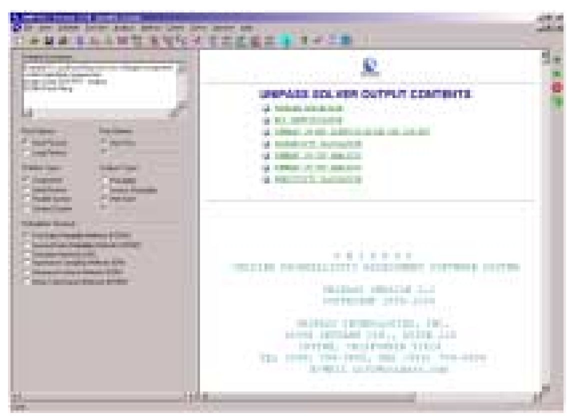

Figure 10 Output File HTML View

15. <u>EXAMPLES</u>

<u>Example 1</u>

For the limit-state function shown as

$$g(x) = \sqrt{\frac{1}{1 + 0.25\left(\frac{x}{0.001}\right)^2}} \qquad (2)$$

where x is a normal distribution with zero as mean and 0.01 as standard deviation. Perform the CDF/PDF analysis by FORM with minimum 5 points in the range of pf = (0.001, 0.999). This is a multi MPP problem and the limit-state function is bounded. By using Modified U-space/X-linearized MPP Identification Method in ***UNIPASS***, a total 240 limit-state functions have been called and 8 points in CDF/PDF curve have been produced as listed in Table 1 and Figure 11.

*Table 1*Results of CDF/PDF of g(x) for Ex. 1

G(x)	PDF	CDF
6.0683508E-02	1.4457447E-07	9.9999419E-04
1.3891935E-01	2.7955831E+00	1.3928117E-01
2.1715518E-01	2.4440711E+00	3.5635768E-01
2.9539102E-01	1.5610073E+00	5.0784692E-01
5.3009853E-01	6.8460610E-01	7.4865997E-01
7.6480605E-01	2.9031485E-01	8.6692960E-01
8.8215980E-01	7.7147441E-01	9.1500039E-01
9.9951356E-01	3.3161930E-09	9.9900001E-01

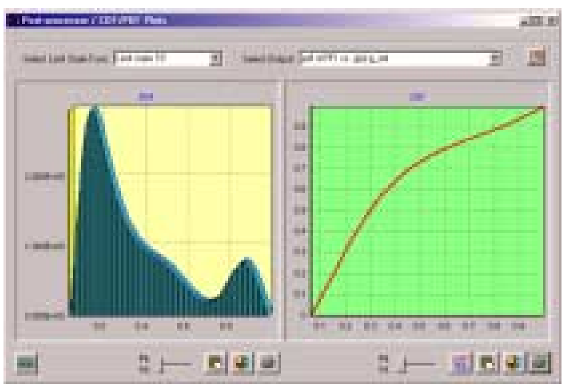

*Figure 11*PDF/CDF Plots for Ex. 1

The results have been verified by using Monte Carlo simulation method with 39548 limit-state function calls to reach the 0.05 COV convergence tolerance. The CDF/PDF plots of g(x) from Monte Carlo simulation are shown in Figure 12.

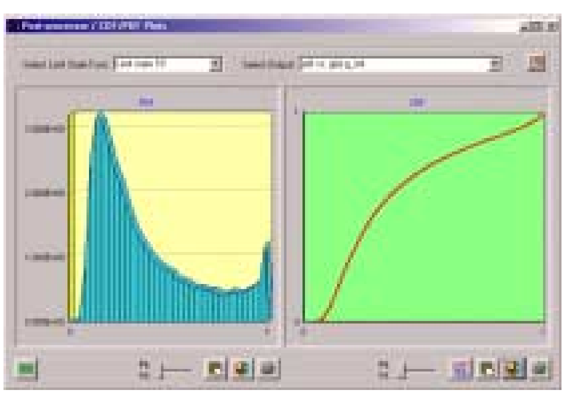

Figure 12 PDF/CDF Plots of Ex. 1 by Monte Carlo Simulation Method

Example 2

Gear contact stress design: Consider the following gear contact stress equation:

$$R = \frac{2T_p E P_d^{\,3} \sin\phi}{(1-\nu^2)\pi\lambda N_1^{\,3}\theta_1 \cos^2\phi\left[\sin\phi - \frac{\theta_1\cos\phi}{(m_g+1)}\right]} \quad (3)$$

where

$$\lambda = f\,\frac{P_d}{N_1}$$

m_g = Gear ratio = 3.78

$$\theta_1 = \text{Rollangle} = \frac{2\left(\left[\left(\frac{N_1}{2}\sin\phi\right)^2 + N_1 + 1\right]^{0.5} - \pi \qquad \cos\phi\right)}{N_1 \qquad \cos\phi}$$

f: Face width (in.)
T_p: Torque (lb/in.)
P_d: Diametrical pitch (1/in.)
N_1: Number of teeth of the pinion
ϕ: Pressure angle (deg.)
E: Modulus of elasticity (psi)
ν: Poisson's ratio

Random Variables:

f ~ Lognormal (μ=0.5, σ=0.025)
T_p ~ Lognormal (μ=108, σ=5.4)
P_d ~ Lognormal (μ=9, σ=0.45)
N_1 ~ Lognormal (μ=18, σ=0.9)
ϕ ~ Lognormal (μ=20, σ=1.0)
E ~ Lognormal (μ=30*10^6, σ=1500000)
ν ~ Lognormal (μ=0.25, σ=0.0125)

where μ is mean and σ is standard deviation. Perform CDF/PDF Analysis for random function, R, with minimum 10 points in the range of Pf = 0.000001~0.999999 by using FORM method.

By using ***UNIPASS***, there are 14 points resulting from 563 limit-state function calls. The First-order PDF/CDF of R are listed in Table 2 and the corresponding CDF/PDF plot is given in Figure 13.

*Table 2*Results of CDF/PDF of R for Ex. 2

R	*PDF*	*CDF*
2.2451890E+09	1.237682E-14	9.999919E-07
2.6308153E+09	4.851332E-13	5.5639865E-05
3.0164417E+09	6.052410E-12	9.6847354E-04
3.4020681E+09	3.417369E-11	7.5740708E-03
3.9805077E+09	1.658078E-10	5.9875786E-02
4.5589473E+09	3.502791E-10	2.1066676E-01
5.1373869E+09	4.236824E-10	4.4255955E-01

5.7158265E+09	3.478096E-10	6.7106849E-01
6.8727057E+09	1.103704E-10	9.2691163E-01
8.0295848E+09	1.912395E-11	9.8931123E-01
9.1864640E+09	2.375226E-12	9.9878212E-01
1.0343343E+10	2.457275E-13	9.9987936E-01
1.1500222E+10	2.311875E-14	9.9998887E-01
1.2657102E+10	2.086106E-15	9.9999900E-01

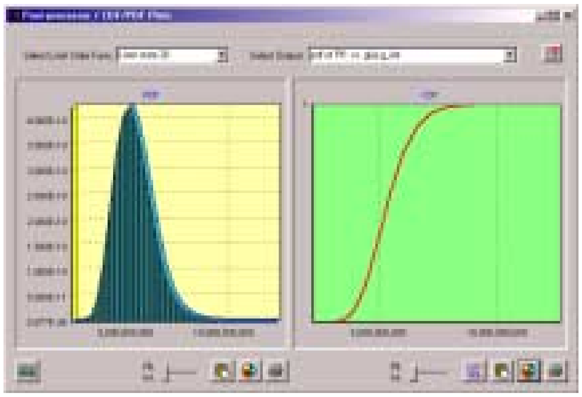

*Figure 13*PDF/CDF Plot of Ex. 2

Up to 28 sensitivity measurements and the corresponding graphic views are available in this analysis. As an example, the sensitivity measurement of $(\text{standard deviation})\times\left\{\frac{\partial pf_1}{\partial \text{mean}}\right\}$ for all the 14 levels of random function *R* (as shown in the legend) and all the random variables (as shown in X-axis) given in Figure 14.

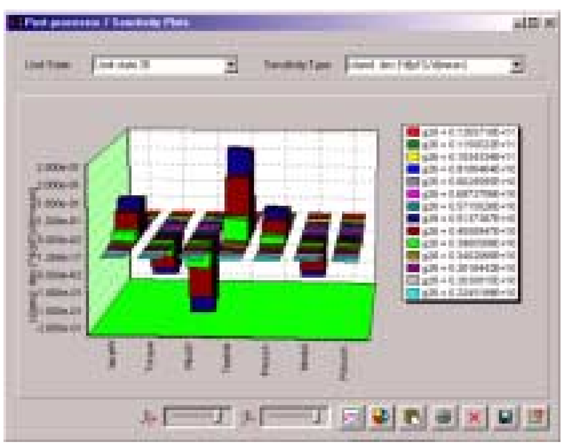

Figure 14 Example of a Sensitivity Plot

By clicking an icon in the preceding sensitivity plot, the 3-D plot can be projected into 2-D plot where the levels of random function are given in X-axis and random variables are given in the legend as shown in Figure 15. This 2-D plot shows the change of sensitivity measurement along with the values of random function.

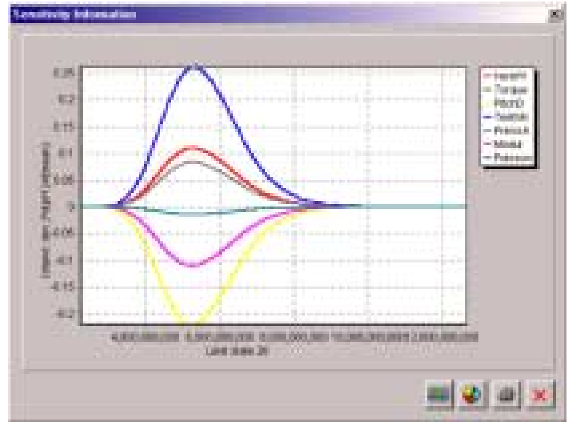

Figure 15 The projection view of the sensitivity plot shown in *Figure 14*

For any selected level of random function, the sensitivity plot can be obtained by clicking the value of the random function shown in the legend in Figure 14. For example, for the random function R = 3.0164417E+09, the sensitivity of $(\text{standard deviation})\times\left\{\frac{\partial pf_1}{\partial \text{mean}}\right\}$ is given in Figure *16*.

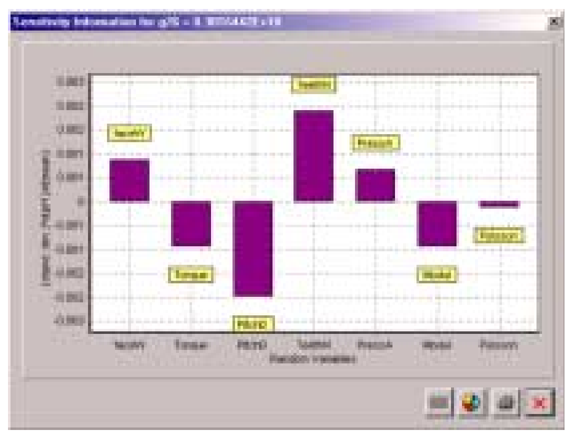

Figure 16 A sensitivity plot for a selected random function value

For any random variable, the sensitivity verse the levels of random function is available by clicking on the corresponding series in *Figure 14*. For example, for random variable, N_1: *Number of teeth of the pinion*, the sensitivity plot verse the levels of random function R is given in *Figure* 17.

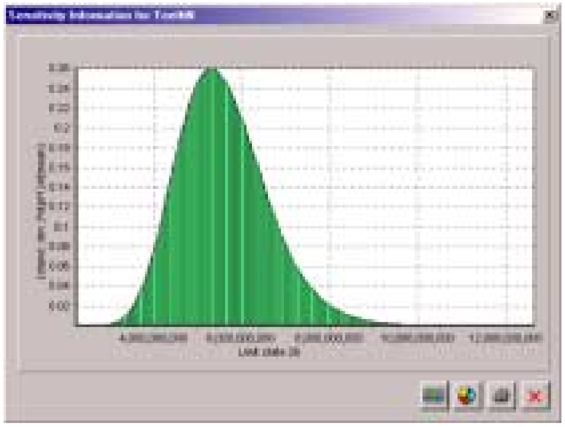

Figure 17 A sensitivity plot with respect to a distribution parameter of a selected random variable

<u>REFERENCES:</u>

1. ***UNIPASS*** Software Specification, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
2. ***UNIPASS*** Quick Start Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
3. ***UNIPASS*** Output Reference Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
4. ***UNIPASS*** Distribution Library Reference Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
5. ***UNIPASS*** NASTRAN.IN Reference Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
6. ***UNIPASS*** CUST.IN Reference Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.
7. ***UNIPASS*** UNIPASS.IN Reference Guide, Version 4.0, PredictionProbe, Inc., Newport Beach, California, June, 2001.

Table 3 — ***UNIPASS*** problem type specification

Analysis / **Problem**	**Probability Analysis**		**Inverse Analysis**		**PDF/CDF Analysis**	
Component Problem	FORM SORM SM	ISM RSM MVBM	FORM SM ISM	RSM MVBM	FORM SORM SM	ISM RSM MVBM
Serial Problem	FORM SORM SM	ISM RSM MVBM				
Parallel Problem	FORM SORM SM	ISM RSM MVBM				
General Problem	FORM SORM SM	ISM RSM MVBM				